 

**ИНЖЕНЕРНАЯ ГЕОМЕТРИЯ И КОМПЬЮТЕРНАЯ ГРАФИКА. ЦИФРОВАЯ ПОДДЕРЖКА ЖИЗНЕННОГО ЦИКЛА ИЗДЕЛИЙ/ENGINEERING GEOMETRY AND COMPUTER GRAPHICS. DIGITAL PRODUCT LIFECYCLE SUPPORT**

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# STRUCTURAL RELATIONS BETWEEN DISTRIBUTIVE AND CHAINED HESITANT FUZZY MODULES

Research article

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**Abstract**

This paper explores the structural relationship between distributive and chained hesitant fuzzy modules over rings under the tools of level submodule and homomorphic image. Some new theoretical results are derived about preserving distributive behavior under hesitant fuzzy epimorphisms and the importance of the chain conditions in the determination of distributive behavior. It is demonstrated that any chained hesitant fuzzy module is distributive, but this is not generally true for the converse if the corresponding level submodules are not ordered in an additional way. Additionally, equivalences between chained hesitant fuzzy modules and their levels structures are obtained. The study also shows that the distributive properties of the level modules of a hesitant fuzzy module completely characterize the distributivity of the module. A number of explicit examples are provided over integer modules showing that, for some of them, the chain condition is not satisfied, but the property of distributivity remains true for their construction. The results give an algebraic deeper insight into the hesitant fuzzy module structures and will help to understand the difference between distributive and chained properties in hesitant fuzzy algebraic systems.

**Keywords:** hesitant fuzzy module, distributive module, chained module, level submodule.

# СТРУКТУРНЫЕ СООТНОШЕНИЯ МЕЖДУ ДИСТРИБУТИВНЫМИ И ЦЕПОЧЕЧНЫМИ НЕЧЕТКИМИ МОДУЛЯМИ С КОЛЕБЛЮЩЕЙСЯ НЕОПРЕДЕЛЕННОСТЬЮ

Научная статья

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**Аннотация**

В данной статье исследуется структурная взаимосвязь между дистрибутивными и цепочечными нечеткими модулями с колеблющейся неопределенностью над кольцами с использованием инструментов уровневого подмодуля и гомоморфного образа. Получены новые теоретические результаты о сохранении дистрибутивного поведения при нечетких эпиморфизмах с колеблющейся неопределенностью, а также о важности цепочечных условий при определении дистрибутивного поведения. Показано, что любой цепочечный нечеткий модуль с колеблющейся неопределенностью является дистрибутивным, однако обратное утверждение в общем случае не выполняется, если соответствующие уровневые подмодули не упорядочены дополнительным образом. Кроме того, получены эквивалентности между цепочечными нечеткими модулями с колеблющейся неопределенностью и их уровневыми структурами. Исследование также показывает, что дистрибутивные свойства уровневых модулей нечеткого модуля с колеблющейся неопределенностью полностью характеризуют дистрибутивность самого модуля. Приведен ряд наглядных примеров на целочисленных модулях, показывающих, что для некоторых из них цепочечное условие не выполняется, однако свойство дистрибутивности сохраняется в их конструкции. Полученные результаты дают более глубокое алгебраическое понимание структур нечетких модулей с колеблющейся неопределенностью и помогают понять различие между дистрибутивными и цепочечными свойствами в нечетких алгебраических системах с колеблющейся неопределенностью.

**Ключевые слова:** нечеткий модуль с колеблющейся неопределенностью, дистрибутивный модуль, цепочечный модуль, уровневый подмодуль.

## Introduction

The theory of hesitant fuzzy sets has been emerging as a crucial extension of classical fuzzy sets, since it allows to model uncertainty due to several potential membership values. Since the inception it has found a lot of interest in various mathematical and applied fields, especially in algebraic structures where uncertainty and vagueness abound naturally. In this class of structures, hesitant fuzzy modules are considered as a flexible structure to research the generalized module properties under the context of the hesitant membership environment.

Distributive and chained modules are basic classes of modules in module theory which have important roles to play in the understanding of lattice structure of submodules and decomposition behavior. The above ideas extend to the uncertain fuzzy setting, providing new opportunities for analysis of algebraic systems that are influenced by uncertainty. There are still not enough studies in the literature about the structural relationship between distributive HFMs and chained HFMs.

This paper is dedicated to establishing new relationships between these two families of modules with hesitations, by level subset techniques and homomorphic characterizations. A few sufficient conditions for the distributivity are deduced, especially with the chain structure of level modules. Further, the preservation of distributivity under hesitant fuzzy epimorphisms is also explored. Based on equivalence results of chained hesitant fuzzy modules with their level submodules, the paper further develops the results.

For purposes of illustration, concrete examples to illustrate the theoretical results based on direct sums of integer modules are created. The above examples show that distributivity strictly weaker than chained property in hesitant fuzzy setting, particularly by direct sum construction. Thus, the results of this study come to a better understanding of the structure of the hesitant fuzzy modules and can serve as a basis for further study in hesitant fuzzy algebra and its applications and these results are an extension of the research work reported in the sources [1], [3], [5], [6].

**Main results**

**2.1. Theorem 1**

Let $\mathscr{F}$ and $\mathcal{G}$ be hesitant fuzzy $\mathscr{R}$-modules on $\mathscr{R}$-modules $\mathscr{M}$ and $\mathscr{N}$, respectively, and let $f : \mathscr{M} \to \mathscr{N}$ be an $\mathscr{R}$-epimorphism. Assume that every hesitant f-submodule of $\mathscr{F}$ is $\mathscr{F}$-invariant. If $\mathscr{F}$ is a distributive hesitant fuzzy $\mathscr{R}$-module, then $f(\mathscr{F})$ likewise constitutes a distributive H.F. $\mathscr{R}$-module on $\mathscr{N}$.

*Proof:*

Let $\mathscr{G}$, $\mathscr{H}$, and $\mathscr{I}$ be hesitant fuzzy submodules of $f(\mathscr{F})$. The definition of inverse image enables construction of hesitant fuzzy submodules through the sets $f^{-1}(\mathscr{G})$, $f^{-1}(\mathscr{H})$, and $f^{-1}(\mathscr{I})$, all residing within the framework of $\mathscr{F}$. The distributive hesitant fuzzy $\mathscr{R}$-module structure of $\mathscr{F}$ permits establishment of the equality:

$$f^{-1}(\mathcal{G}) \cap \left(f^{-1}(\mathscr{H}) + f^{-1}(\mathscr{H})\right) = \left(f^{-1}(\mathcal{G}) \cap f^{-1}(\mathscr{H})\right) + \left(f^{-1}(\mathscr{G}) \cap f^{-1}(\mathscr{F})\right)$$

The properties of inverse images under fuzzy homomorphisms enable derivation of:

$$f^{-1}(\mathcal{G} \cap (\mathscr{H} + \mathscr{I})) = f^{-1}(\mathcal{G} \cap \mathscr{H}) + f^{-1}(\mathcal{G} \cap \mathscr{I})$$

Applying *f* to both sides combined with the surjective property of *f* and the fact that every hesitant fuzzy submodule of $\mathscr{F}$ remains $\mathscr{F}$-invariant leads to the deduction that:

$$f\left(f^{-1}(\mathscr{G} \cap (\mathscr{H} + \mathscr{F}))\right) = f\left(f^{-1}(\mathscr{G} \cap \mathscr{H})\right) + f\left(f^{-1}(\mathscr{G} \cap \mathscr{F})\right)$$

Consequently, the expression $\mathscr{G} \cap (\mathscr{H} + \mathscr{I}) = (\mathscr{G} \cap \mathscr{H}) + (\mathscr{G} \cap \mathscr{I})$ holds true. Thus $f(\mathscr{F})$ demonstrates adherence to the distributive property, thereby establishing itself as a distributive hesitant fuzzy $\mathscr{R}$-module.

**2.2. Theorem 2**

Every chained hesitant fuzzy $\mathscr{R}$-module qualifies as a distributive hesitant fuzzy $\mathscr{R}$-module.

*Proof:*

Let $\mathscr{F}$ be a chained hesitant fuzzy $\mathscr{R}$-module and let $\mathscr{G}$, $\mathscr{H}$, $\mathscr{I}$ be hesitant fuzzy submodules of $\mathscr{F}$. Since $\mathscr{F}$ is chained, the family $\{\mathscr{G}, \mathscr{H}, \mathscr{I}\}$ is totally ordered under inclusion. Without loss of generality, assume $\mathscr{G} \subseteq \mathscr{H} \subseteq \mathscr{I}$. Then we obtain:

$$\mathcal{G} \cap (\mathscr{H} + \mathscr{I}) = \mathcal{G} \cap \mathscr{I} = \mathcal{G}, \quad \text{given that } \mathscr{H} + \mathscr{I} = \mathscr{I}.$$

On the other hand:

$$(\mathcal{G} \cap \mathscr{H}) + (\mathcal{G} \cap \mathscr{I}) = \mathcal{G} + \mathcal{G} = \mathcal{G}.$$

Thus, $\mathcal{G} \cap (\mathscr{H} + \mathscr{I}) = (\mathcal{G} \cap \mathscr{H}) + (\mathcal{G} \cap \mathscr{I})$, which demonstrates that $\mathscr{F}$ exhibits distributivity.

**2.3. Theorem 3**

Let $\mathscr{F}$ be a hesitant fuzzy $\mathscr{R}$-module. If for every $j \in (0, 1]$, the level module $\mathscr{F}j$ constitutes a chained $\mathscr{R}$-module, then $\mathscr{F}$ forms a distributive hesitant fuzzy $\mathscr{R}$-module.

*Proof:*

Take $j \in (0, 1]$. By hypothesis, $\mathscr{F}j$ is a chained $\mathscr{R}$-module. Hence, for any submodules $\mathscr{G}j$, $\mathscr{H}j$, $\mathscr{I}j$ of $\mathscr{F}j$, we have:

$$\mathcal{G}_j \cap (\mathscr{H}_j + \mathscr{I}_j) = (\mathcal{G}_j \cap \mathscr{H}_j) + (\mathcal{G}_j \cap \mathscr{I}_j)$$

Thus, $\mathscr{F}j$ is a distributive $\mathscr{R}$-module for all $j \in (0, 1]$. By the level characterization of distributive hesitant fuzzy modules, it follows that $\mathscr{F}$ qualifies as a distributive hesitant fuzzy $\mathscr{R}$-module.

**2.3.1 Proposition 1**

Let $\mathscr{F}$ be a distributive H.F. $\mathscr{R}$-module. If for every $j \in (0, 1]$, the lattice of submodules of $\mathscr{F}j$ is linearly ordered, then $\mathscr{F}$ constitutes a chained hesitant fuzzy $\mathscr{R}$-module.

*Proof:*

Let $\mathscr{G}$ and $\mathscr{H}$ be hH.F. submodules of $\mathscr{F}$. Then for each $j \in (0, 1]$, the level subsets $\mathscr{G}j$ and $\mathscr{H}j$ are submodules of $\mathscr{F}j$. Since the lattice of submodules of $\mathscr{F}j$ is linearly ordered, either $\mathscr{G}j \subseteq \mathscr{H}j$ or $\mathscr{H}j \subseteq \mathscr{G}j$. This holds for all $j \in (0, 1]$. Hence, by the characterization of hesitant fuzzy inclusion via level subsets, either $\mathscr{G} \subseteq \mathscr{H}$ or $\mathscr{H} \subseteq \mathscr{G}$. Therefore, $\mathscr{F}$ qualifies as a chained H.F. $\mathscr{R}$-module.

**2.4. Theorem 4**

Let $\mathscr{F}$ denote a hesitant fuzzy module over the ring $\mathscr{R}$. The subsequent statements will demonstrate their equivalence to one another:

1. $\mathscr{F}$ is a chained hesitant fuzzy $\mathscr{R}$-module.
2. For every $j \in (0, 1]$, $\mathscr{F}j$ is a chained $\mathscr{R}$-module.

*Proof:*

 

(1) ⇒ (2): Assume $\mathscr{F}$ is chained. Let $\mathscr{G}j$, $\mathscr{H}j$ be submodules of $\mathscr{F}j$. Then there exist hesitant fuzzy submodules $\mathscr{G}$, $\mathscr{H}$ such that $\mathscr{G}j$ and $\mathscr{H}j$ are their level subsets. Since $\mathscr{F}$ is chained, either $\mathscr{G} \subseteq \mathscr{H}$ or $\mathscr{H} \subseteq \mathscr{G}$, hence $\mathscr{G}j \subseteq \mathscr{H}j$ or $\mathscr{H}j \subseteq \mathscr{G}j$. Thus $\mathscr{F}j$ is chained.

(2) ⇒ (1): Assume each $\mathscr{F}j$ is chained. Let $\mathscr{G}$, $\mathscr{H}$ be hesitant fuzzy submodules of $\mathscr{F}$. Then for all $j \in (0, 1]$, either $\mathscr{G}j \subseteq \mathscr{H}j$ or $\mathscr{H}j \subseteq \mathscr{G}j$. Hence, $\mathscr{G} \subseteq \mathscr{H}$ or $\mathscr{H} \subseteq \mathscr{G}$. Therefore, $\mathscr{F}$ is chained.

The preceding results demonstrate that the chained property is stronger than distributivity in the hesitant fuzzy setting, while distributivity alone does not guarantee the chained condition unless additional structure (such as linear ordering at each level) is imposed.

**2.5. Theorem 5**

Let $\mathscr{F}$ be a hesitant f. $\mathscr{R}$-module on an $\mathscr{R}$-module $\mathscr{M}$. Assume that for every level $\alpha \in (0, 1]$, the level submodule $\mathscr{F}\alpha$ constitutes a chained $\mathscr{R}$-module. Then $\mathscr{F}$ is a distributive H.F $\mathscr{R}$-module.

*Proof:*

Assume that for every $\alpha \in (0, 1]$, the level submodule $\mathscr{F}\alpha$ is chained. Since every chained module exhibits distributivity, it follows that for all $\alpha \in (0, 1]$:

$$\mathcal{G}\alpha \cap (\mathscr{H}\alpha + \mathscr{I}\alpha) = (\mathcal{G}\alpha \cap \mathscr{H}\alpha) + (\mathcal{G}\alpha \cap \mathscr{I}\alpha)$$

Let $\mathscr{G}, \mathscr{H}, \mathscr{I}$ be hesitant fuzzy submodules of $\mathscr{F}$. Then:

$$(\mathcal{G} \cap (\mathscr{H} + \mathscr{D}))\alpha = \mathcal{G}\alpha \cap (\mathscr{H}\alpha + \mathscr{H}\alpha) \ , \ ((\mathcal{G} \cap \mathscr{H}) + (\mathcal{G} \cap \mathscr{D}))\alpha = (\mathcal{G}\alpha \cap \mathscr{H}\alpha) + (\mathcal{G}\alpha \cap \mathscr{H}\alpha)$$

Given that $\mathscr{F}\alpha$ is distributive, we obtain:

$$(\mathcal{G} \cap (\mathscr{H} + \mathscr{D}))\alpha = ((\mathcal{G} \cap \mathscr{H}) + (\mathcal{G} \cap \mathscr{D}))\alpha$$

Thus:

$\mathcal{G} \cap (\mathscr{H} + \mathscr{I}) = (\mathcal{G} \cap \mathscr{H}) + (\mathcal{G} \cap \mathscr{I})$ . Hence $\mathscr{F}$ qualifies as a distributive hesitant fuzzy $\mathscr{R}$-module.

**2.6. Corollary 1**

If $\mathscr{F}$ is a distributive hesitant fuzzy $\mathscr{R}$-module and for every $\alpha \in (0, 1]$, $\mathscr{F}\alpha$ is Noetherian, then $\mathscr{F}\alpha$ is chained for all $\alpha \in (0, 1]$.

*Proof:*

Assume $\mathscr{F}$ is distributive and each $\mathscr{F}\alpha$ is Noetherian. Then each $\mathscr{F}\alpha$ constitutes a distributive $\mathscr{R}$-module. A classical result implies that every Noetherian distributive module decomposes into chained components. Since level submodules inherit minimal structure from the hesitant fuzzy setting, each $\mathscr{F}\alpha$ must be chained.

**2.7. Example 1**

Let $\mathscr{R} = \mathbb{Z}$, and consider the $\mathbb{Z}$-modules $\mathscr{M}_1 = \mathbb{Z}$ and $\mathscr{M}_2 = \mathbb{Z}$. Define $\mathscr{M} = \mathscr{M}_1 \oplus \mathscr{M}_2$. Let $\mathscr{F}_1$ and $\mathscr{F}_2$ be hesitant fuzzy $\mathscr{R}$-modules on $\mathscr{M}_1$ and $\mathscr{M}_2$, respectively, defined as follows:

$$\mathscr{F}_1(x) = \begin{cases} \{1, 0.9\}, & \text{if } x \in 2\mathbb{Z} \\ \{0.6\}, & \text{otherwise} \end{cases}$$

$$\mathscr{F}_2(y) = \begin{cases} \{1, 0.8\}, & \text{if } y \in 3\mathbb{Z} \\ \{0.4\}, & \text{otherwise} \end{cases}$$

Define the hesitant fuzzy direct sum $\mathscr{F} = \mathscr{F}_1 \oplus \mathscr{F}_2$ on $\mathscr{M}$ by:

$$\mathscr{F}(x, y) = \min\left\{\mathscr{F}_1(x), \mathscr{F}_2(y)\right\}$$

Now we analyze the level subsets. For $\alpha \in (0, 1]$, we have: $\mathscr{F}\alpha = (\mathscr{F}_1)\alpha \oplus (\mathscr{F}_2)\alpha$. Observe that:

$$\left(\mathscr{F}_1\right)\alpha = \{\mathbb{Z}, 0 < \alpha \le 0.6; 2\mathbb{Z}, 0.6 < \alpha \le 0.9; 2\mathbb{Z}, 0.9 < \alpha \le 1\}$$

$$\left(\mathscr{F}_2\right)\alpha = \{\mathbb{Z}, 0 < \alpha \le 0.4; 3\mathbb{Z}, 0.4 < \alpha \le 0.8; 3\mathbb{Z}, 0.8 < \alpha \le 1\}$$

$$\mathscr{F}\alpha = \{\mathbb{Z} \oplus \mathbb{Z}, 0 < \alpha \le 0.4; \mathbb{Z} \oplus 3\mathbb{Z}, 0.4 < \alpha \le 0.5; 2\mathbb{Z} \oplus 3\mathbb{Z}, 0.5 < \alpha \le 1\}$$

Now note:

· $\mathbb{Z}$, $2\mathbb{Z}$, and $3\mathbb{Z}$ are chained $\mathbb{Z}$-modules.

· ann( $\mathscr{M}_1$ ) = ann( $\mathscr{M}_2$ ) = {0}, hence ann( $\mathscr{M}_1$ ) + ann( $\mathscr{M}_2$ ) = $\mathbb{Z}$.

By the classical result concerning direct sums of distributive modules, each level $\mathscr{F}\alpha$ constitutes a distributive module. Therefore, $\mathscr{F}$ qualifies as a distributive hesitant fuzzy $\mathscr{R}$-module. However, note that $\mathscr{F}\alpha$ is generally not chained (since direct sums of chained modules are not necessarily chained).

This illustration demonstrates that distributivity of hesitant fuzzy modules is strictly weaker than the chained property under direct sum constructions. Thus, the direct sum structure preserves distributivity but not the chain condition, highlighting the depth of the main theorem.

The preceding example shows that while the chain condition guarantees distributivity, the converse fails under direct sum constructions. This reveals a structural gap between hesitant fuzzy chained and distributive modules.

**2.8. Example 2**

Let $\mathscr{R} = \mathbb{Z}$ and consider the $\mathscr{R}$-modules:

$$\mathscr{M}_1 = \mathbb{Z} \ , \ \mathscr{M}_2 = \mathbb{Z} \ , \ \mathscr{M} = \mathscr{M}_1 \oplus \mathscr{M}_2 \ .$$

Define two hesitant fuzzy $\mathscr{R}$-modules $\mathscr{F}_1$ and $\mathscr{F}_2$ on $\mathscr{M}_1$ and $\mathscr{M}_2$, respectively, as follows:

$$\mathscr{F}_1(x) = \{\{1, 0.9, 0.8\}, \text{ if } x \in 2\mathbb{Z}; \{0.6, 0.5\}, \text{ otherwise } \}.$$

$$\mathscr{F}_2(y) = 1, 0.85, \text{ ify } \in 3\mathbb{Z}; 0.7, 0.4, \text{ otherwise}$$

Define the hesitant fuzzy direct sum $\mathscr{F} = \mathscr{F}_1 \oplus \mathscr{F}_2$ on $\mathscr{M}$ by:

$$\mathscr{F}(x, y) = \min\{\mathscr{F}_1(x), \mathscr{F}_2(y)\}.$$

**Step 1:** Level structure. For each $\alpha \in (0, 1]$, we have $\mathscr{F}\alpha = (\mathscr{F}_1)\alpha \oplus (\mathscr{F}_2)\alpha$. By direct computation:

$$\left(\mathscr{F}_1\right)\alpha = \{\mathbb{Z}, 0 < \alpha \le 0.5; 2\mathbb{Z}, 0.5 < \alpha \le 0.9; 2\mathbb{Z}, 0.9 < \alpha \le 1\}$$

$$\left(\mathscr{F}_2\right)\alpha = \{\mathbb{Z}, 0 < \alpha \le 0.4; 3\mathbb{Z}, 0.4 < \alpha \le 0.85; 3\mathbb{Z}, 0.85 < \alpha \le 1\}$$

$$\mathscr{F}\alpha = \{\mathbb{Z} \oplus \mathbb{Z}, 0 < \alpha \le 0.4; \mathbb{Z} \oplus 3\mathbb{Z}, 0.4 < \alpha \le 0.5; 2\mathbb{Z} \oplus 3\mathbb{Z}, 0.5 < \alpha \le 1\}$$

**Step 2:** Distributivity. Each of the modules $\mathbb{Z}$, $2\mathbb{Z}$, and $3\mathbb{Z}$ is a distributive $\mathbb{Z}$-module. Moreover:

$$\operatorname{ann}(\mathscr{M}_1) = \operatorname{ann}(\mathscr{M}_2) = 0 \ , \ \operatorname{ann}(\mathscr{M}_1) + \operatorname{ann}(\mathscr{M}_2) = \mathbb{Z} \ .$$

Hence, by the classical result on direct sums of distributive modules, each level module $\mathscr{F}\alpha$ is distributive. Therefore, $\mathscr{F}$ is a distributive hesitant fuzzy $\mathscr{R}$-module.

**Step 3:** Failure of the chain condition. Consider the following submodules of $\mathscr{M}$: $\mathscr{G} = 2\mathbb{Z} \oplus 0$, $\mathscr{H} = 0 \oplus 3\mathbb{Z}$. Then: $\mathscr{G} \nsubseteq \mathscr{H}$ and $\mathscr{H} \nsubseteq \mathscr{G}$. Thus, $\mathscr{M}$ is not a chained $\mathscr{R}$-module. Consequently, for suitable hesitant fuzzy submodules corresponding to $\mathscr{G}$ and $\mathscr{H}$, the inclusion relation fails, and hence $\mathscr{F}$ is not a chained H.F $\mathscr{R}$-module.

**Conclusion**

In this paper, several fundamental properties of distributive and chained hesitant fuzzy modules were investigated through the study of level submodules and hesitant fuzzy homomorphisms. The obtained results established that every chained hesitant fuzzy module is distributive, while distributivity alone is generally insufficient to guarantee the chain condition without additional structural assumptions. Moreover, distributivity was shown to be fully characterized by the distributive behavior of the associated level modules.

The study also demonstrated that distributive hesitant fuzzy modules remain stable under suitable epimorphic images and that the structure of level submodules plays a decisive role in determining both distributive and chained properties. Furthermore, explicit direct sum examples clarified the structural gap between distributive and chained hesitant fuzzy modules by showing that distributivity may be preserved even when the chain condition fails.

The results presented in this paper contribute to enhancing the theoretical understanding of hesitant fuzzy module structures and provide more links between classical module theory and hesitant fuzzy algebra. The obtained results can be used as a starting point for future studies on decomposition theory, homological properties and generalized fuzzy algebraic systems.

**Конфликт интересов**

Не указан.

**Рецензия**

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**Conflict of Interest**

None declared.

**Review**

All articles are peer-reviewed. But the reviewer or the author of the article chose not to publish a review of this article in the public domain. The review can be provided to the competent authorities upon request.

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